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Mathematical Modelling And Simulation of Quadrotor

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Abstract: *Nowadays unmanned aerial vehicles are being used surveilance, search and rescue, damage assesment alongwith areas where dangereous for people to work. In this work, 4-rotor unmanned aerial vehicle design, altitude and orientation controls of unmanned aerial vehicle are aimed. Quadrotor's mathematical model is mostly focused in this work. Quadrotor has 6 degrees of freedom. Inertial Measurement Unit will be composed for flight stability. Quadrotor's altitude, speed, direction and position datas will be chased and evaluated by the Inertial Measurement Unit. There will be a Quadrotor's central microcontroller which will manage all the components that necessary for flight. This microcontroller will use the data coming from Inertial Measurement Unit and user inorder to control motor speed.*
The Quadrotor of mathematical model were obtained using Newton-Euler equation. The PID controller is designed to control the model. The performance of controllers and the model was investigated by using Matlab Simulink program.
Keywords:*Inertia,Matlab,Newton-Euler, PID,Quadrotor,Simulink*

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I. Introduction

The Quadrotor is an unmanned aerial vehicle that is placed diagonally and is driven by four engines at its ends. The Quadrotor has four motors in the front, rear, right and left, with the motors moving and propelling forces in the direction of the axis of rotation. The propellers on the front and rear rotate counterclockwise, while the propellers on the left and right rotate clockwise. When all of the propellers rotate at equal speeds, the total torque at the center of the Quadrotor is balanced, so that the angle of rotation around its axis is constant [1]. If the velocities of the left and right propellers are different from each other, there is a difference between the lift forces and the yaw angle changes. Likewise, when the speeds of the front and rear propellers are different from each other, the pitch of the angle changes. The Quadrotor moves in the z-axis direction by changing the speeds of all the propellers in the same direction. The speed of the propellers moving in the same direction is changed according to the speed of the two propellers rotating in the other direction, allowing the Quadrotor to rotate around its axis [2]. The direction of rotation of the propellers, the lifting forces and rotation angles of these rotations are shown in Figure 1.

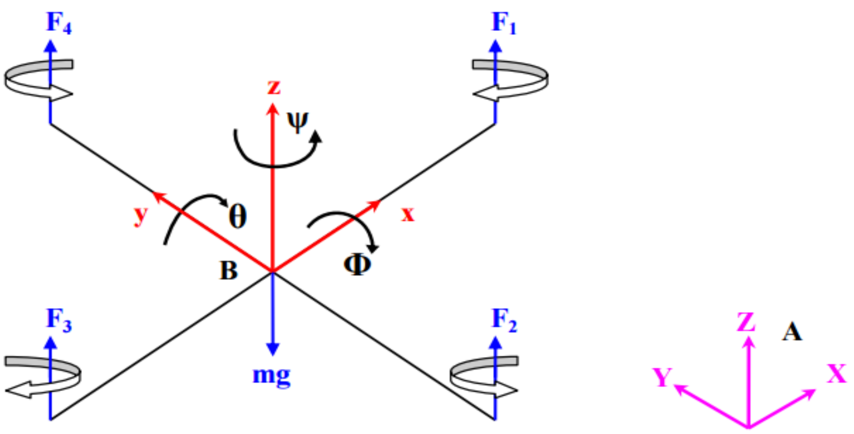

Figure 1. Quadrotor axes and forces

In figure 1, the Quadrotor's body fixed frame "B" and inertia frame "A"; yaw (Φ, roll), pitch (θ, pitch) and orientation (ψ, yaw) angles around the x, y, and z axes; The main forces affecting the Quadrotor are "F1, F2, F3, F4, mg" and the directions of rotation of the four propellers.

II. Mathematıcal Model

2.1. Configuration of Quadrotor

An important aspect of the mathematical model of Quadrotor is the coordinate system used in the system. One of the configurations shown with "+" or "X" can be used in the coordinate systems shown below:

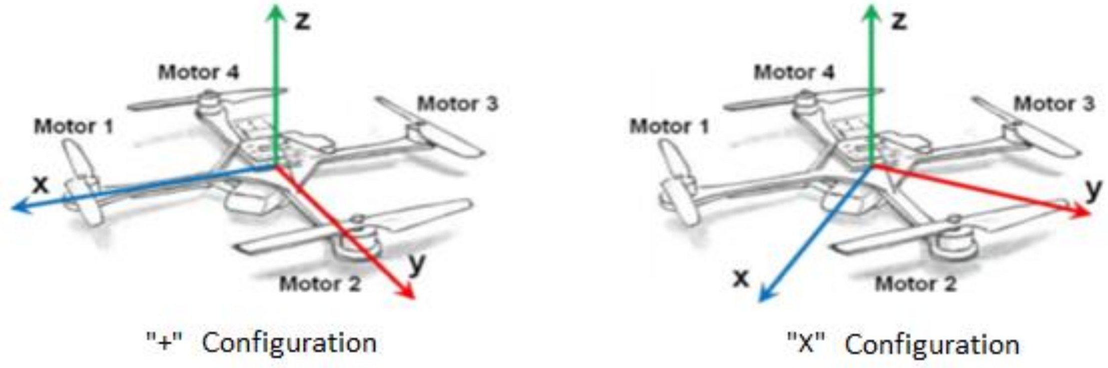

Figure 2."+" ve "*X*" Configuration Diagram

The used "+" configuration is that the arms of the motor 1 extend in the same direction as the x axis, the motor 2 is located in the y axis, and the upward direction of the z axis is shown in Figure 2 [3].

2.2. Mass Moment of Inertia Matrix

Inertia matrix is defined inertia mass moment which is placed between the specified axes of the Quadrotors. With some approaches, in order to be constructed the inertia matrixthe moment of inertia between the x, y, and z axes can be determined. When the "+" or "X" configuration is used, the inertia matrix is as shown below.

$$J^b = \begin{bmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & 0 \\ 0 & 0 & J_{zz} \end{bmatrix} \quad (1)$$

J^b: Quadrotor's inertiaaccording to body frame with J_{xx}, J_{yy} and J_{zz}

J_{xx}, J_{yy} *and* J_{zz}: Quadrotor's inertia at three axis

2.3. Thrust Coefficient

The thrust of the motors is the driving force behind all the Quadrotor maneuvers. Thus, controlling design and simulation is seen as part of the integration. The thrust (T) provided by a single motor / propeller system is calculated using the following formula [4].

$$T = C_T \rho A_r r^2 \varpi^2 \quad (2)$$

CT: thrust coefficient for a given rotor
p: the density of the air
Ar: Rotational cross-sectional area of the propeller
R: Rotor radius
ɯ: Angular velocity of the rotor

2.4. Torque Coefficient

The torque strength of the motor / propeller system must be determined in order to recognize the motor influences in the yaw (ψ). This can be done in a manner similar to that of thrust tests. The block parameter equation for the torque coefficient is shown below.

$$Q = c_Q \varpi^2 \quad (3)$$

Q: Torque generated by the motor
CQ: Torque coefficient for motor / propeller system
This torque provides a moving force for orientation in the z-axis system.

2.5. Initial Matrix Structure

Analysis programs using the data obtained at the end of each test can calculate the coefficients needed to identify the system. With the help of these coefficient, the following matrix can be created ("+" configuration) which defines thrusts and torques.

$$\begin{bmatrix} \Sigma T \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} c_T & c_T & c_T & c_T \\ 0 & d_+ c_T & 0 & -d_+ c_T \\ -d_+ c_T & 0 & d_+ c_T & 0 \\ -c_Q & c_Q & -c_Q & c_Q \end{bmatrix} \begin{bmatrix} \varpi_1^2 \\ \varpi_2^2 \\ \varpi_3^2 \\ \varpi_4^2 \end{bmatrix} \quad (4)$$

d is the distance between the motor and the axis of rotation, and d + is the distance from the center of the Quadrotor to the motor / propeller. In the "X" configuration, d + sin (45) can be used instead of dx. Because this value is the distance between the axis of the motor / impeller and the axis of rotation. C_T effect; it is distributed between 4 motors for pitching and rolling so that C_Q does not change in this configuration setting.

$$\begin{bmatrix} \Sigma T \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} c_T & c_T & c_T & c_T \\ -d_x c_T & d_x c_T & d_x c_T & -d_x c_T \\ -d_x c_T & -d_x c_T & d_x c_T & d_x c_T \\ -c_Q & c_Q & -c_Q & c_Q \end{bmatrix} \begin{bmatrix} \varpi_1^2 \\ \varpi_2^2 \\ \varpi_3^2 \\ \varpi_4^2 \end{bmatrix} \qquad (5)$$

2.6. Throttle Command Relation

The thrust and torque coefficients are based on the relationship of the motors to the RPM. Accordingly, the throttle command is converted to RPM values with a linear equation. The following equation was established for this purpose:

$$\varpi_{ss} = (\%Throttle)c_R + b \qquad (6)$$

ɯss: motor speed (RPM)

% Throttle: The percent power of the battery for the Quadrotor's takeoff

C_R: RPM conversion factor corresponding to% Throttle value for RPM conversion

b: Point of linear regression relation to y axis

2.7. Gyroscopic Forces

Gyroscopic forces are a phenomenon that occurs when a rotating body changes its rotation axis. Gyroscopic forces are added to form the forces before the moment matrix is formed. The gyroscopic forces appearing on the body are determined by the speed of each motor / propeller system (xi), together with the components (Jm), pitching and yawing degree (P and Q) and inertia of each motor. The gyroscopic torques produced by the finishing motors of the pitching and Rolling motions are shown below [4]:

$$\tau_{\theta_{gyro}} = J_m P \left(\frac{\pi}{30}\right)(-\varpi_1 + \varpi_2 - \varpi_3 + \varpi_4) \qquad (7)$$

$$\tau_{\phi_{gyro}} = J_m Q \left(\frac{\pi}{30}\right)(\varpi_1 - \varpi_2 + \varpi_3 - \varpi_4) \qquad (8)$$

π / 30: transition from RPM to radian which must occur for the gyroscopic force to be calculated.

2.8. Result Matrix Structure

When aerodynamic, gyroscopic and thrust moment forces are added, equations in the form of a matrix for simulation can be obtained. For the "+" configuration, the result matrix is created by Quadrotor engine / propeller systems.

$$M_{A,T}^b = \begin{bmatrix} d_+ c_T \varpi_2^2 - d_+ c_T \varpi_4^2 + J_m Q(\frac{\pi}{30})(\varpi_1 - \varpi_2 + \varpi_3 - \varpi_4) \\ -d_+ c_T \varpi_1^2 + d_+ c_T \varpi_3^2 + J_m P(\frac{\pi}{30})(-\varpi_1 + \varpi_2 - \varpi_3 + \varpi_4) \\ -c_Q \varpi_1^2 + c_Q \varpi_2^2 - c_Q \varpi_3^2 + c_Q \varpi_4^2 \end{bmatrix} (9)$$

The body frame generated by the aerodynamics, thrusts and torques generated in the $M^b_{A,T}$system expresses the current momentum. The quadrotor's body also specifies the forces of motion of the rotors on lifting and gravity. The lift force can be expressed by the formula shown below.

$$F_{A,T}^b = \begin{bmatrix} 0 \\ 0 \\ c_T(\varpi_1^2 + \varpi_2^2 + \varpi_3^2 + \varpi_4^2) \end{bmatrix} \qquad (10)$$

$F^b_{A,T}$is the force acting on the body frame on the aerodynamic and thrust-connected quadrotor in the positive z direction. After additional research and testing, effects can be added, such as aerodynamic drag frame and rotor lash.

2.9. State Equation

The state equations describing dynamic models are like the formulas shown below. The first thing we will deal with is the Angular Velocity Condition Equation.

$${}^b\dot{\omega}_{b|i}^b = \begin{bmatrix} \dot{P} \\ \dot{Q} \\ \dot{R} \end{bmatrix} = (J^b)^{-1}\left[M_{A,T}^b - \Omega_{b|i}^b J^b \omega_{b|i}^b\right] \qquad (11)$$

${}^b\dot{\omega}_{b|i}^b$: The angular acceleration along each axis in the body frame relative to the inertia reference

Eylemsizlik matrisi ve moment matrisleri hesaplandığından dolayı, dönüş hızı için bir vektörel çarpım matrisi olan $\Omega_{b|i}^b$hesaplanabilir.

This equation shows the inertia, the Quadrotor's rolling, the pitching, and the change in yawing rates.

Since the inertia matrix and moment matrices are computed, a vectorial product matrix $\Omega_{b|i}^{b}$ can be calculated for the rotation speed.

$$\Omega_{b|i}^{b} = \begin{bmatrix} 0 & -\mathrm{R} & \mathrm{Q} \\ \mathrm{R} & 0 & -\mathrm{P} \\ -\mathrm{Q} & \mathrm{P} & 0 \end{bmatrix} \qquad (12)$$

The P, Q, and R in this matrix show the rotation degree of the x, y, and z axes, respectively.

$$\omega_{b|i}^{b} = \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \qquad (13)$$

$\omega_{b|i}^{b}$: Rotation of Quadrotor's body in body frame

The next state equation is the Euler kinematic equation which determines the rate of change of the Euler angles in the reference frame of inertia.

$$\dot{\Phi} = H(\Phi)\omega_{b|i}^{b} = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \qquad (14)$$

According to the order of rotation in aviation, it is defined as the rotation of a plane around the z-axis after rotation around y-axis, followed by rotation of the plane about x-axis. Each rotation is based on a right-hand method and a single plane.

Using these three rotations, a joint rotation matrix can be created that can transform the movement of the aircraft into a new reference frame from the body frame. The rotation matrix is transformed from the body frame to the inertia frame and is found using the matrix multiplication. The s, c, and t representations used below are defined as sine, cosine, and tangent functions, respectively.

$$\mathrm{u}^{b} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mathrm{c}(\phi) & \mathrm{s}(\phi) \\ 0 & -\mathrm{s}(\phi) & \mathrm{c}(\phi) \end{bmatrix} \begin{bmatrix} \mathrm{c}(\theta) & 0 & -\mathrm{s}(\theta) \\ 0 & 1 & 0 \\ \mathrm{s}(\theta) & 0 & \mathrm{c}(\theta) \end{bmatrix} \begin{bmatrix} \mathrm{c}(\psi) & \mathrm{s}(\psi) & 0 \\ -\mathrm{s}(\psi) & \mathrm{c}(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathrm{u}^{i} \qquad (15)$$

The inertial rotation matrix in the body frame obtained using the aerial rotation sequence is obtained by matrix multiplication as shown in the following form.

$$C_{b|i} = \begin{bmatrix} \mathrm{c}(\theta)\,\mathrm{c}(\psi) & \mathrm{c}(\theta)\,\mathrm{s}(\psi) & -\mathrm{s}(\theta) \\ (-\,\mathrm{c}(\phi)\,\mathrm{s}(\psi) + \mathrm{s}(\phi)\,\mathrm{s}(\theta)\,\mathrm{c}(\psi)) & (c(\phi)c(\psi) + s(\phi)s(\theta)s(\psi)) & \mathrm{s}(\phi)\,\mathrm{c}(\theta) \\ (s(\phi)s(\psi) + c(\phi)s(\theta)c(\psi)) & (-s(\phi)c(\psi) + c(\phi)s(\theta)s(\psi)) & \mathrm{c}(\phi)\,\mathrm{c}(\theta) \end{bmatrix} \qquad (16)$$

This ZYX series rotation matrix is of particular importance in solving the velocity and position state equations. Using the rotation matrices, the angular velocity of the body frame frame can be correlated with the changes that occur in angle conversion, as shown below. C matrices of Φ and θ are matrices from u^{b}.

$$\omega_{b|i}^{b} = \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + C_{\phi}\left(\begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + C_{\theta}\begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix}\right) \qquad (17)$$

Matrix multiplication and realization of the sum can be found by deriving the Euler Kinematic Equation.

$$\dot{\Phi} = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & t(\theta)\,s(\phi) & t(\theta)\,c(\phi) \\ 0 & c(\phi) & -s(\phi) \\ 0 & s(\phi)\,/c(\theta) & c(\phi)/\,c(\theta) \end{bmatrix} \begin{bmatrix} P \\ Q \\ R \end{bmatrix} = H(\Phi)\omega_{b|i}^{b} \qquad (18)$$

The velocity state equation shows the acceleration in the mass center of the Quadrotor model based on the forces acting on the body and accelerations.

$$^{b}\dot{\mathrm{v}}_{CM|i}^{b} = \left(\frac{1}{m}\right)\mathrm{F}_{A,T}^{b} + g^{b} - \Omega_{b|i}^{b}\omega_{CM|i}^{b} = \begin{bmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \end{bmatrix} \qquad (19)$$

$^{b}\dot{\mathrm{v}}_{CM|i}^{b}$:The linear acceleration of the center of mass in the body frame according to the inertia reference

g^{b}: gravitational acceleration converted into motion in the body frame by the rotation matrix $C_{b|i}$

m: total mass of Quadrotor

$$g^{b} = C_{b|i}g^{i} \qquad (20)$$

Using this equation, the linear acceleration of the Quadrotor in x, y and z directions of the body frame can be calculated.

The Position State equation shows the linear velocity of the mass center of the Quadrotor at the reference of inertia.

$$^{i}\dot{\mathrm{P}}_{CM|i}^{i} = C_{i|b}\mathrm{v}_{CM|i}^{b} = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} \qquad (21)$$

$\mathrm{v}_{CM|i}^{b}$: The velocity of the quadrotor in the body frame rotated to the inertia reference using $C_{i|b}$, which is the transpose of $C_{b|i}$

III. Modelıng Of Mathematıcal Model In Matlab Sımulınk Envıronment

3.1. Purpose

In this research, it is aimed to simulate the Simulink environment and to operate and fly the dynamic performance of the Quadrotor with the simulation program. Several scenarios were developed to model the behavior of quadrotor. During this process, effects such as wind and rain have been neglected.

3.2 Running Simulation

Stages of running this model in MATLAB Simulink environment are shown below.

1. Simulink model of Quadrotor (status command model) does not have a system control to track the position of the index. Instead of this control, only state (φ, θ, ψ) and altidute (Z) commands are monitored using the PID controller. When the model is opened for the first time, a screen as shown in Figure 3 is displayed [5].

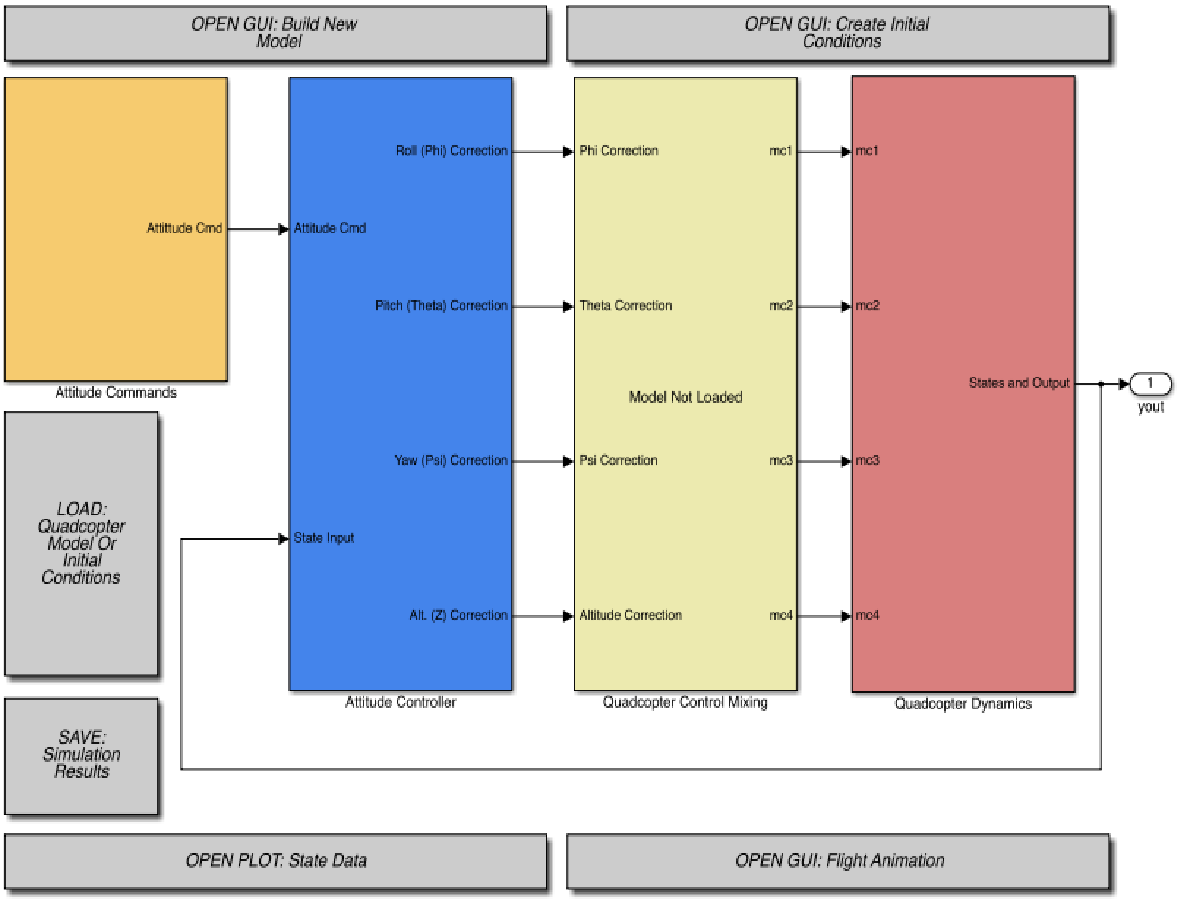

Figure 3. Simulation Overview

2. Double-clicking on the gray blocks, which are buttons, opens the MATLAB Gui windows [6].
3. To run the simulation, double-click the LOAD button. In the download window that opens, first select the file that specifies the initial conditions and install it. The uploaded file contains data showing angular speed, Euler angles, engine speeds, shift speeds and positions in the world. Click on the same button again to load a model for Quadrotor. A second file is then loaded containing the data showing the motors, elongation, arms, weights, and lengths [6].
4. The Attitude Cmd block is opened by double clicking and the signal source blocks are visible. In this simulation, the units of length entered must be meters and the units of the angles must be radians. The next block is the Attitude Controller block and its contents are shown in Figure 4.

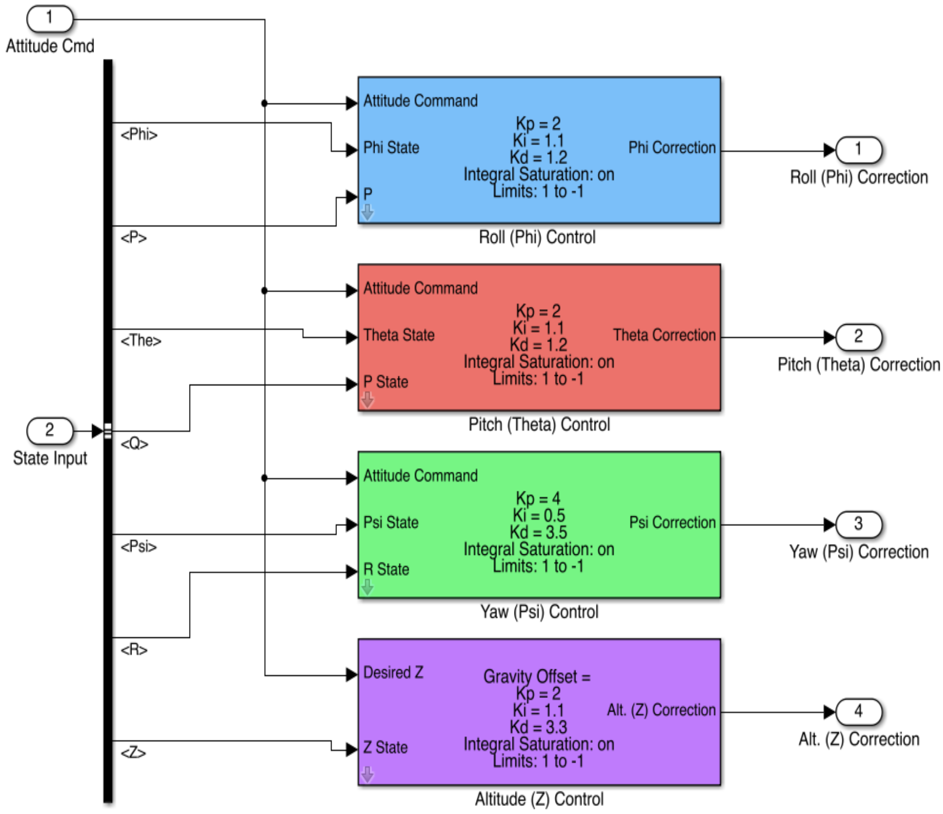

Figure 4. Attitude Controller Detailed display of the block

5. The text in front of these blocks is changed to reflect the parameter values of the mask. To change the PID gain values, one of the blocks can be changed by clicking one of them. [7].
6. After this block the Quadrotor Control Mixer comes. This block sends a correction to each of the Phi, Teta, Psi, Z, and mixes by sending the correct motor to the correct motor. When "Configuration +" appears on the front of this block, the "+" configuration model is loaded. When this block is clicked, the structures shown in Figure4 are found.

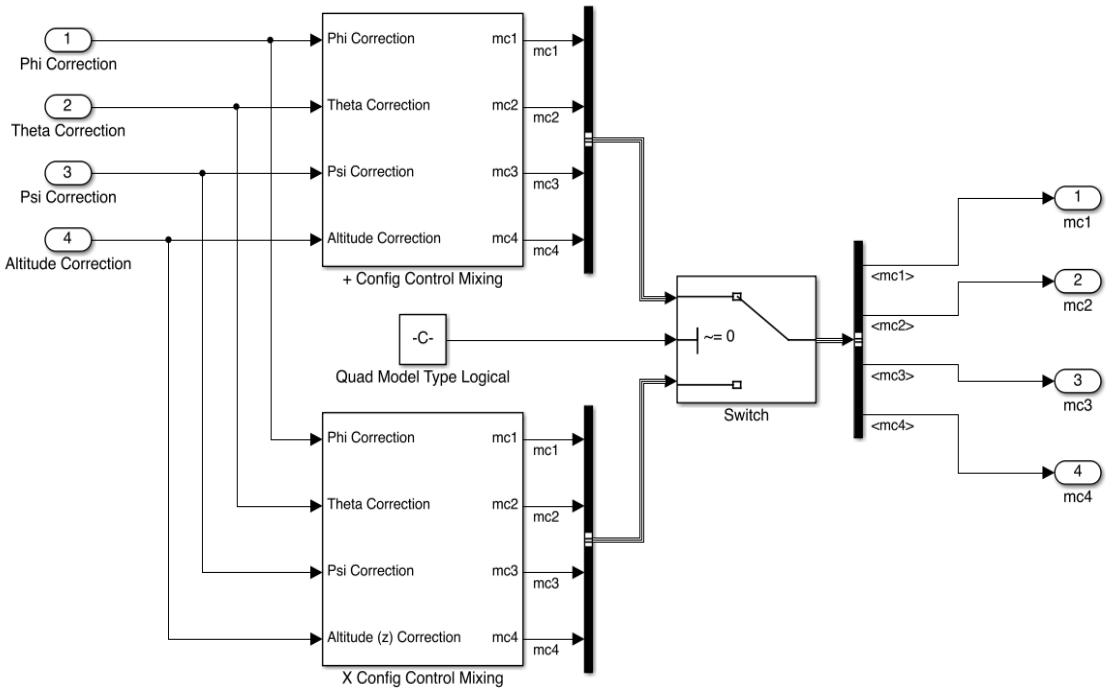

Figure 5. Quadrotor Control Mixer Overview.

7. The purpose of the switch mechanism is to define both the "X" configuration and the "+" configuration in a single block without using another block. Both the "X" configuration and the "+" configuration are shown in Figures 5 and 6and the equation forming each reference output is written [7].

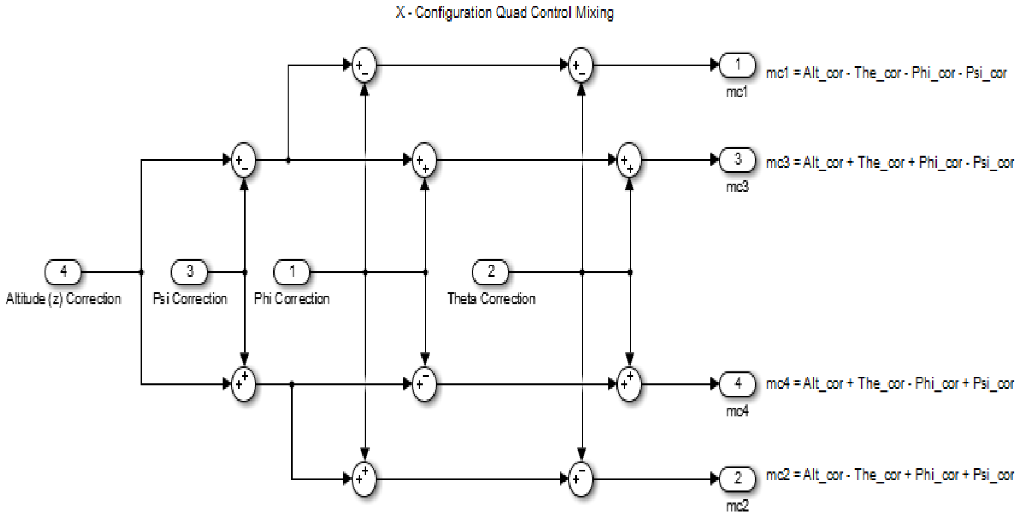

Figure 6. "X" Configuration Control Mixer

8. It should be noted that these output signals are% throttle. The structure of the Quadrotor Dynamics block is shown in Figure 7.
9. The structure of the Quadrotor Dynamics block is shown in Figure 7.

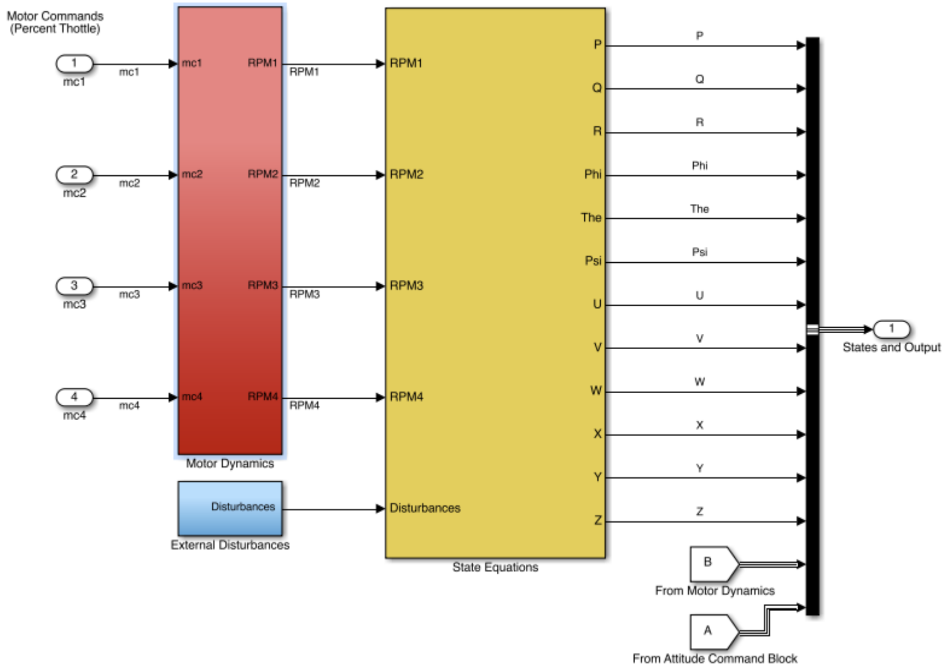

Figure 7. Quadrotor Dynamics Block

10. The input commands of the motor dynamic block are limited between 0% and 100% throttle. Motor cutoff behavior is simulated in very low throttle.
11. When an external disturbing effect is applied to the simulation, the disturbing block is used. Otherwise only all disruptive entries are left as zero.
12. The State Equations Block adds a Level 2 S-Function code written in the MATLAB language. This code is the place where the state equations describing the dynamic behavior of the Quadrotor are executed and can be seen by clicking on the cursor above the block.

13. In order to run the simulation, first the "quadModel" and "IC" constructions are loaded and the simulation is run.
14. Simulation output can be reached by clicking on the OPEN PLOT button. When this button is clicked, the blue, red or green lines show Quadrotor's behavioral simulation responses while the dotted black lines in the first graph show the status and elevation commands. The other graph shows the motor command and speed (RPM) [7].
15. Finally, when the flight animation (OPEN GUI) button is clicked, simulation of a prototype for the Quadrotor can be seen with the data loaded in the simulation.

IV. Conclusıon

In this work the mathematical model of the Quadrotor was obtained and simulated using PID controllers in Matlab Simulink environment. In this simulation the actual parameter values of the Quadrotor are used. By using these values, Quadrotor's behavior in computer environment was examined and it was observed how Quadrotor reacted against parameter changes. In order to the Quadrotor to move more stable, the effect was observed by changing the *Kp*,*Ki* and *Kd* values of the PID controller. Failure to set the flight control parameters of the Quadrotor sufficiently well causes unwanted accidents during flight. Due to the high cost of the hardware units being used and the long time it takes to get them, it is very important to set the parameters of the controller correctly in this type aircraft.

References

[1]. B. Erginer,2007. *Quadrotor VTOL Aracının Modellenmesi ve Kontrolü*, Yüksek Lisans Tezi, İstanbul Teknik Üniversitesi.

[2]. B. Erginer ve E. Altuğ, 2007. *Modeling and PD Control of a Quadrotor VTOL Vehicle*, 2007 IEEE Intelligent Vehicles Symposium Publication.

[3]. Çiçekdemir, Ç., Kesler, M., Karakuzu, C., Yüzgeç, U., ARM Mikrodenetleyici Tabanlı Bilkopter' in 9DOF ile Dengelenmesi, ELECO, Bursa,673-677, 29 Kasım- 1 Aralık, 2012.

[4]. Altın, C., Dört Rotorlu İnsansız Hava Aracının Yükseklik ve Konum Kontrolü, Yüksek Lisans Tezi, Bozok Üniversitesi, Yozgat, 2013.

[5]. R. Mahony, V. Kumar, and P. Corke, "*Multirotor Aerial Vehicles: Modeling, Estimation, and Control of a Quadrotor,*" in IEEE Robotics and Automation Magazine, vol. 19, Sept. 2012, pp. 20-32.

[6]. MathWorks. Simulink: Simulation and Model-Based Design. Web. 14 Mar 2014. http://www.mathworks.com/products/simulink. (Date of access : 10.04.2016).

[7]. David(dch33), "*Quadrotor Dynamic Modeling and Simulation*", Web. 13 Nisan 2015. https://github.com/dch33/Quad-Sim. (Date of access: 17.05.2016).

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